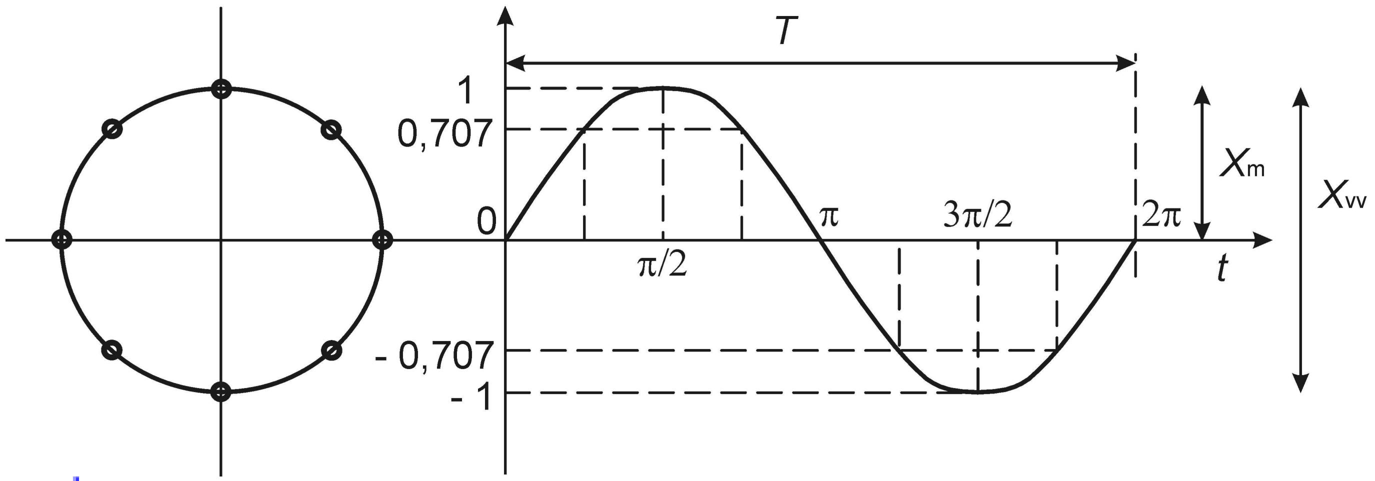
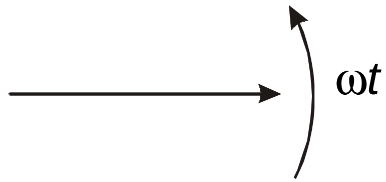
IZMJENIČNE STRUJE

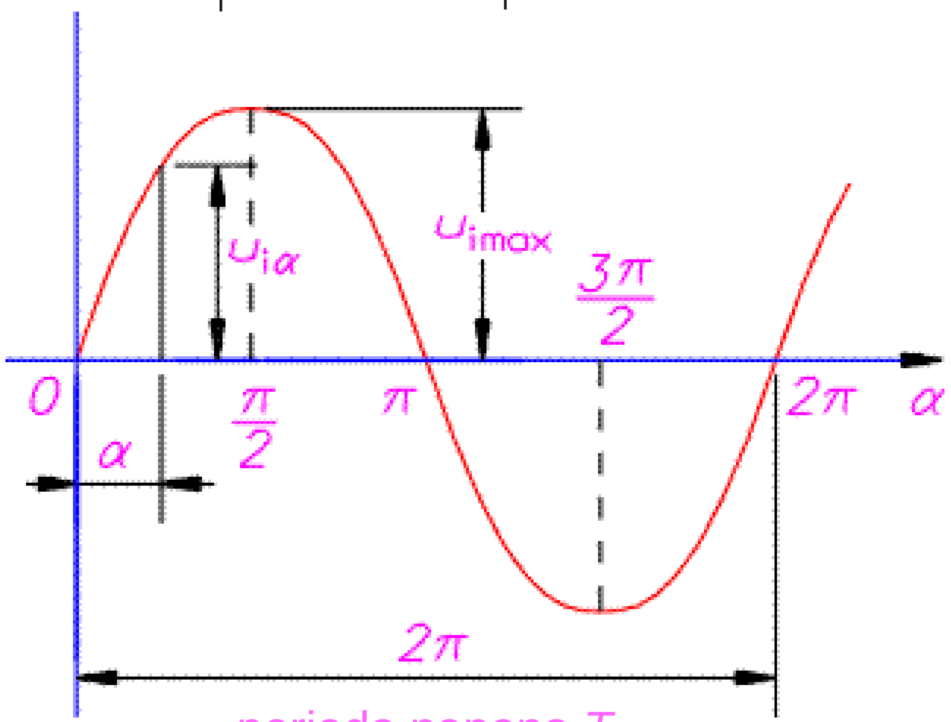


$$f = \frac{1}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$X_m = \frac{X_{vv}}{2}$$





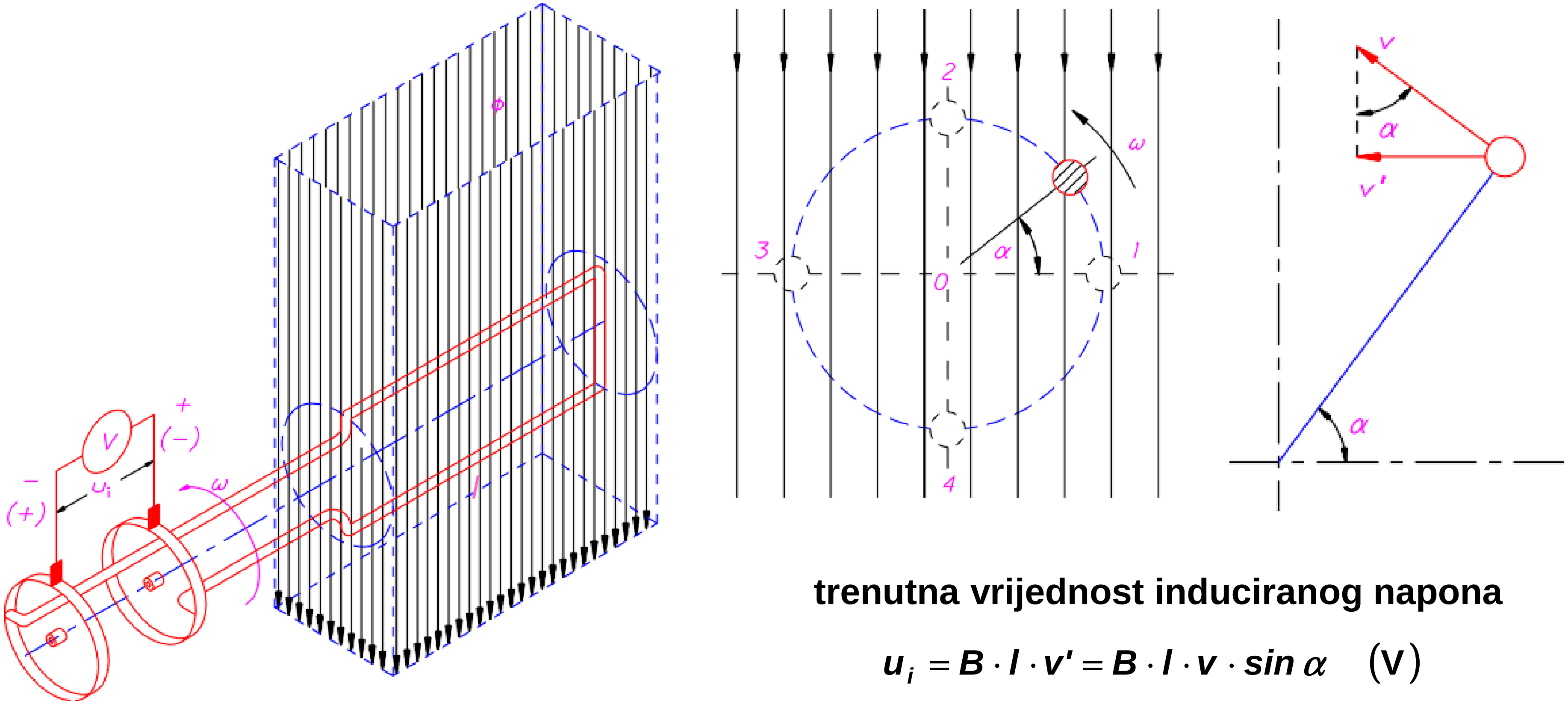
perioda napona *T*

uz kutnu brzinu $\omega = \frac{\alpha}{t}$

kut je $\alpha = \omega \cdot t$

$$u_i = U_{i\,max} \cdot \sin \omega t$$

Dobivanje sinusoide



trenutna vrijednost induciranog napona

$$u_i = B \cdot l \cdot v' = B \cdot l \cdot v \cdot \sin\alpha \quad (\mathrm{V})$$

uz $\alpha = \dfrac{\pi}{2}$ $\quad \sin\alpha = 1$ $\quad$ **i** $\quad U_{i\,max} = B \cdot l \cdot v$

općenito vrijedi $\quad u_i = U_{i\,max} \cdot \sin\alpha$

Efektivna vrijednost sinusoide

rad za stalnu jakost struje $A = I^2 \cdot R \cdot t$

rad za promjenjivu jakost struje $A = \int_o^t i^2 \cdot R \cdot dt$

za jednak rad u oba slučaja ⇒ efektivna vrijednost izmjenične struje (I)

$$I_{ef} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} \qquad I(t) = I_m \sin \omega t$$

$$I_{ef} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T (I_m \sin \omega t)^2 dt} = \sqrt{\frac{I_m^2}{T}\int_0^T \sin^2 \omega t \cdot dt}$$

$$\int \sin^2 ax\, dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax$$

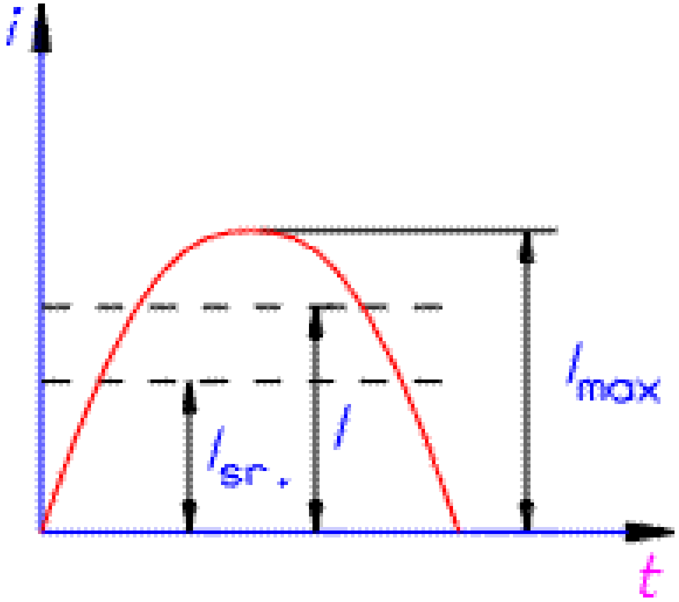
$$I_{ef} = \sqrt{\frac{I_m^2}{T}\left(\frac{1}{2}t - \frac{1}{4\omega}\sin 2\omega t\right)\Bigg|_0^T} =$$

$$I_{ef} = \sqrt{\frac{I_m^2}{T}\left(\frac{1}{2}\cdot T - \underbrace{\frac{1}{4\omega}\sin 2\cdot\frac{2\pi}{T}\cdot T}_{0} - \underbrace{\frac{1}{2}\cdot 0}_{0} + \underbrace{\frac{1}{4\omega}\sin 2\cdot\frac{2\pi}{T}\cdot 0}_{0}\right)} =$$

$$I_{ef} = \sqrt{\frac{I_m^2}{T}\cdot\frac{T}{2}} = \frac{I_m}{\sqrt{2}} \qquad I_{ef} = \frac{I_m}{\sqrt{2}}$$

odnosno za bilo koju sinusnu veličinu: $X_{ef} = \frac{X_m}{\sqrt{2}}$

karakteristične vrijednosti izmjenične struje – efektivna i srednja vrijednost



efektivne vrijednosti za sinusni valni oblik

$$I = I_{max} \cdot \frac{1}{\sqrt{2}}$$

$$U = U_{max} \cdot \frac{1}{\sqrt{2}}$$

trenutne vrijednosti sinusnog valnog oblika izražene efektivnim vrijednostima

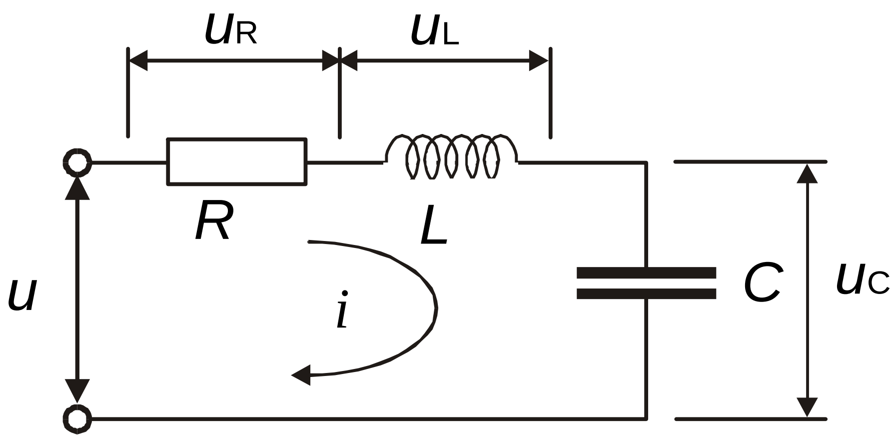
$$i = \sqrt{2} \cdot I \cdot \sin \omega t$$

$$u = \sqrt{2} \cdot U \cdot \sin \omega t$$

temeljem količine naboja određuje se srednja vrijednost za sinusni valni oblik

$$I_{sr} = \frac{2 I_{max}}{\pi} \qquad U_{sr} = \frac{2 U_{max}}{\pi}$$

Fazni pomak



$$u_R = i \cdot R \qquad u_L = L\frac{di}{dt} \qquad u_C = \frac{1}{C}\int i \cdot dt$$

$$u = i \cdot R + L\frac{di}{dt} + \frac{1}{C}\int i \cdot dt$$

Izvor napona sa sinusnim oblikom
=> poteći će struja

$$i = I_{max} \cdot \sin(\omega t)$$

$$u_R = R \cdot I_{max} \sin(\omega t)$$

$$u_L = L\frac{d I_{max} \sin(\omega t)}{dt} = L\frac{d}{dt} I_{max} \sin(\omega t) = L \cdot I_{max} \frac{d}{dt}\sin(\omega t) = \underbrace{\omega L}_{\text{induktivni otpor } X_L} \cdot I_{max} \cos(\omega t)$$

$$u_L = X_L \cdot I_{max} \cos(\omega t)$$

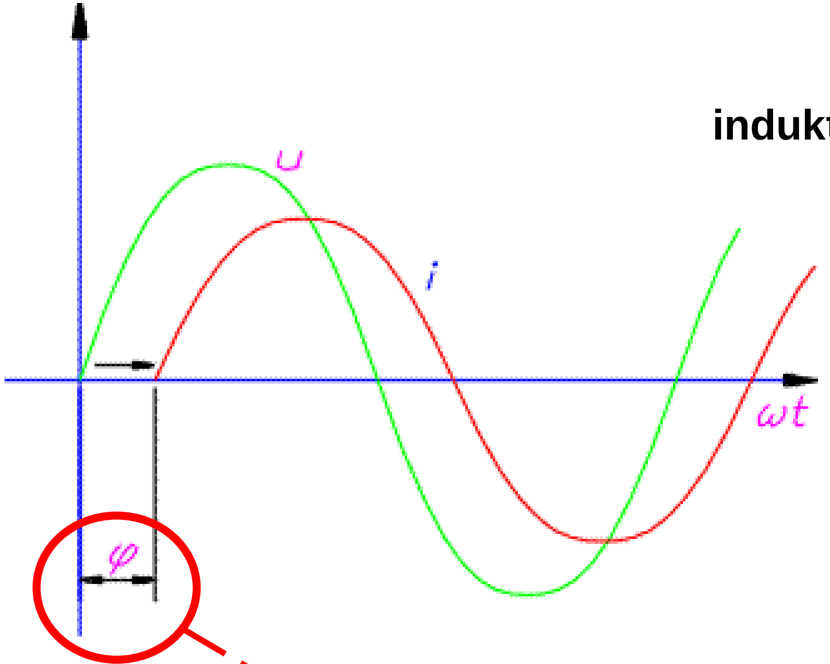
$$u_C = \frac{1}{C}\int I_{max} \sin(\omega t) \cdot dt = \frac{1}{C} I_{max} \int \sin(\omega t) \cdot dt = \underbrace{\frac{1}{\omega \cdot C}}_{\text{kapacitivni otpor } X_C} I_{max} (-\cos(\omega t))$$

$$u_C = X_C \cdot I_{max} (-\cos(\omega t))$$

$$u = R \cdot I_{max} \sin(\omega t) + X_L \cdot I_{max} \cos(\omega t) + X_C \cdot I_{max} (-\cos(\omega t))$$

$$u = R \cdot I_{max} \sin(\omega t) + X_L \cdot I_{max} \sin(\omega t + 90) + X_C \cdot I_{max} \sin(\omega t - 90)$$

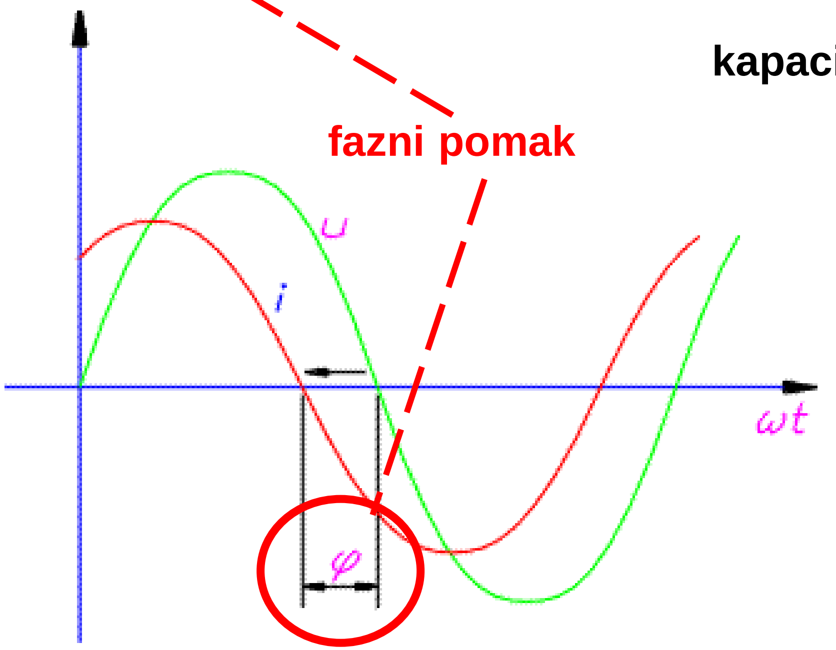
induktivni krug



$$u = \sqrt{2} \cdot U \cdot \sin \omega t$$

$$i = \sqrt{2} \cdot I \cdot \sin(\omega t - \varphi)$$

kapacitivni krug



$$u = \sqrt{2} \cdot U \cdot \sin \omega t$$

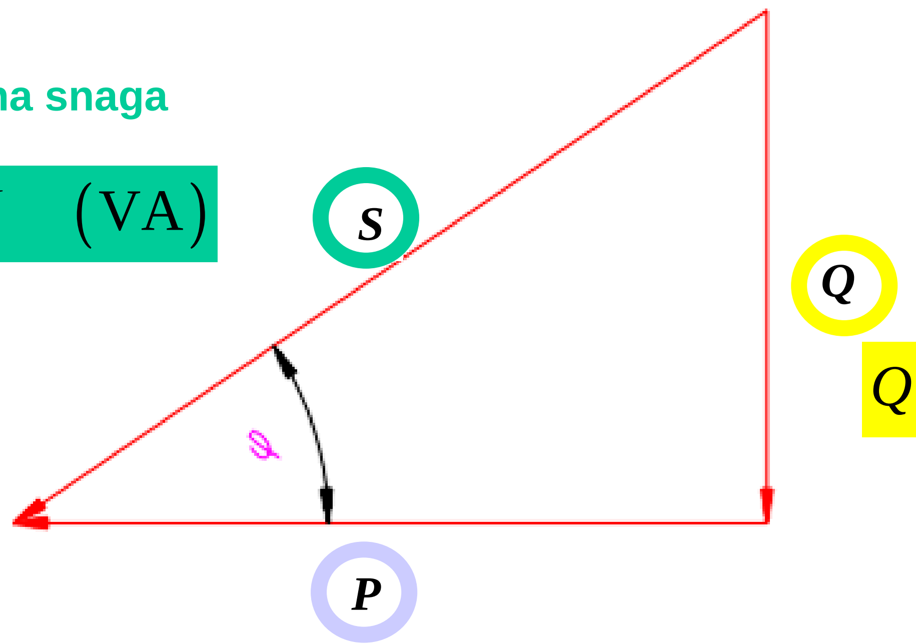
$$i = \sqrt{2} \cdot I \cdot \sin(\omega t + \varphi)$$

Rad i snaga izmjenične struje

rad u vremenu *T* $$A = \int_0^T u \cdot i \cdot dt$$

snaga $$P = \frac{1}{T}\int_0^T u \cdot i \cdot dt$$

$$P = \frac{1}{T}\int_0^T \sqrt{2} \cdot U \cdot \sin \omega t \cdot \sqrt{2} \cdot I \cdot \sin(\omega t - \varphi) \cdot dt$$



prividna snaga

$$S = U \cdot I \quad (\mathrm{VA})$$

jalova snaga

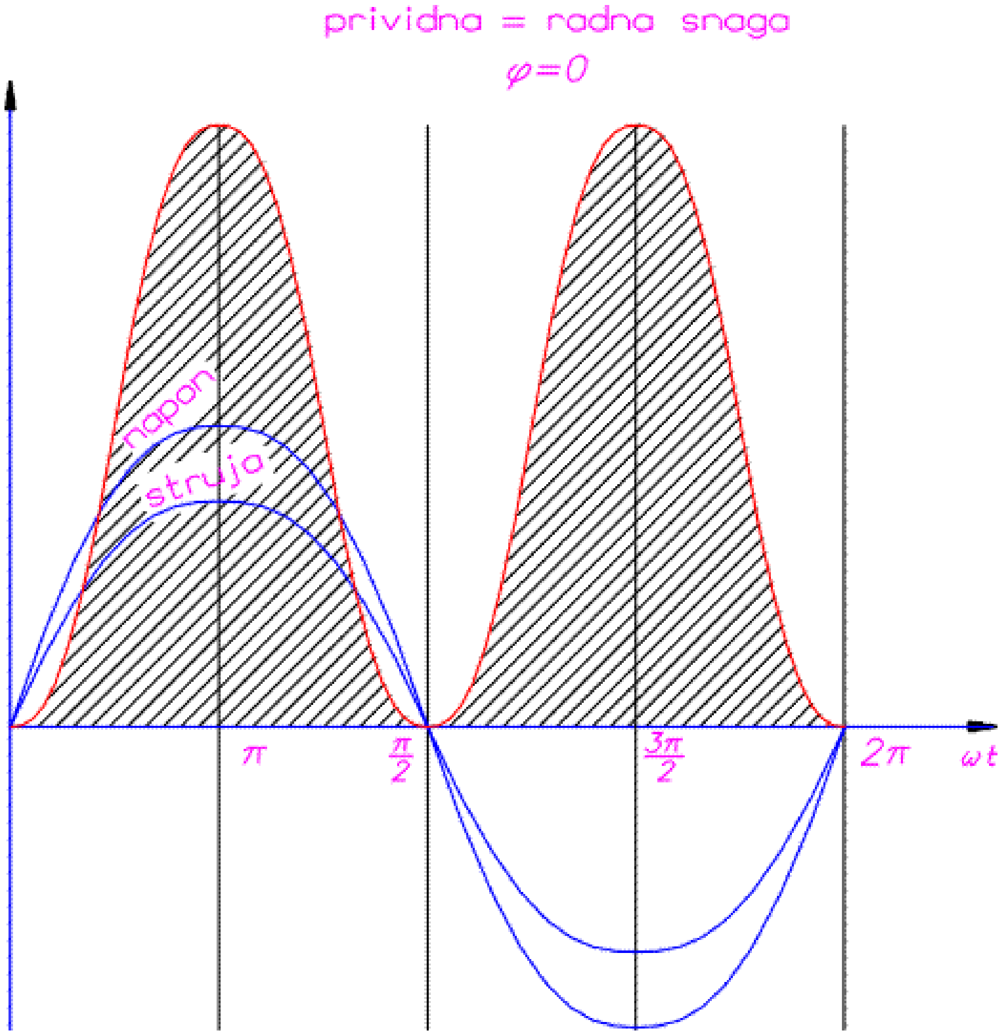
$$Q = U \cdot I \cdot \sin\varphi \quad (\mathrm{VAr})$$

radna snaga

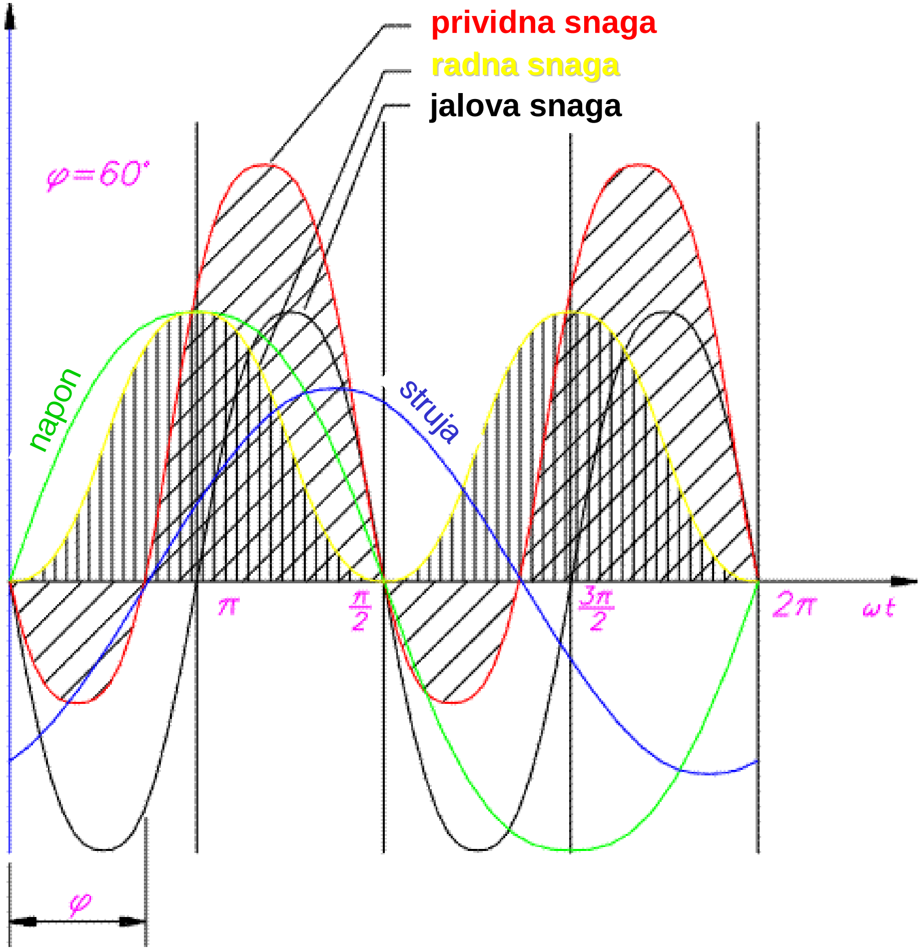
$$P = U \cdot I \cdot \cos\varphi \quad (\mathrm{W})$$

$\cos\varphi$ **- faktor snage**

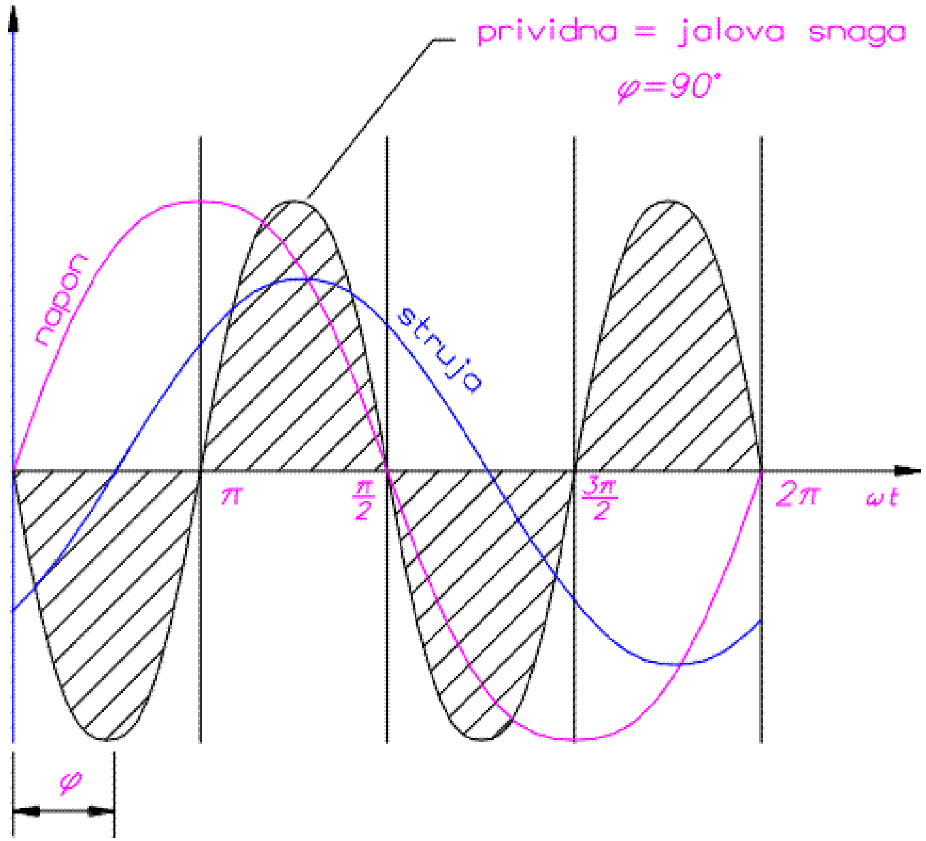
trenutne vrijednosti snage za $\varphi = 0$



trenutne vrijednosti snage za $\varphi = 60°$

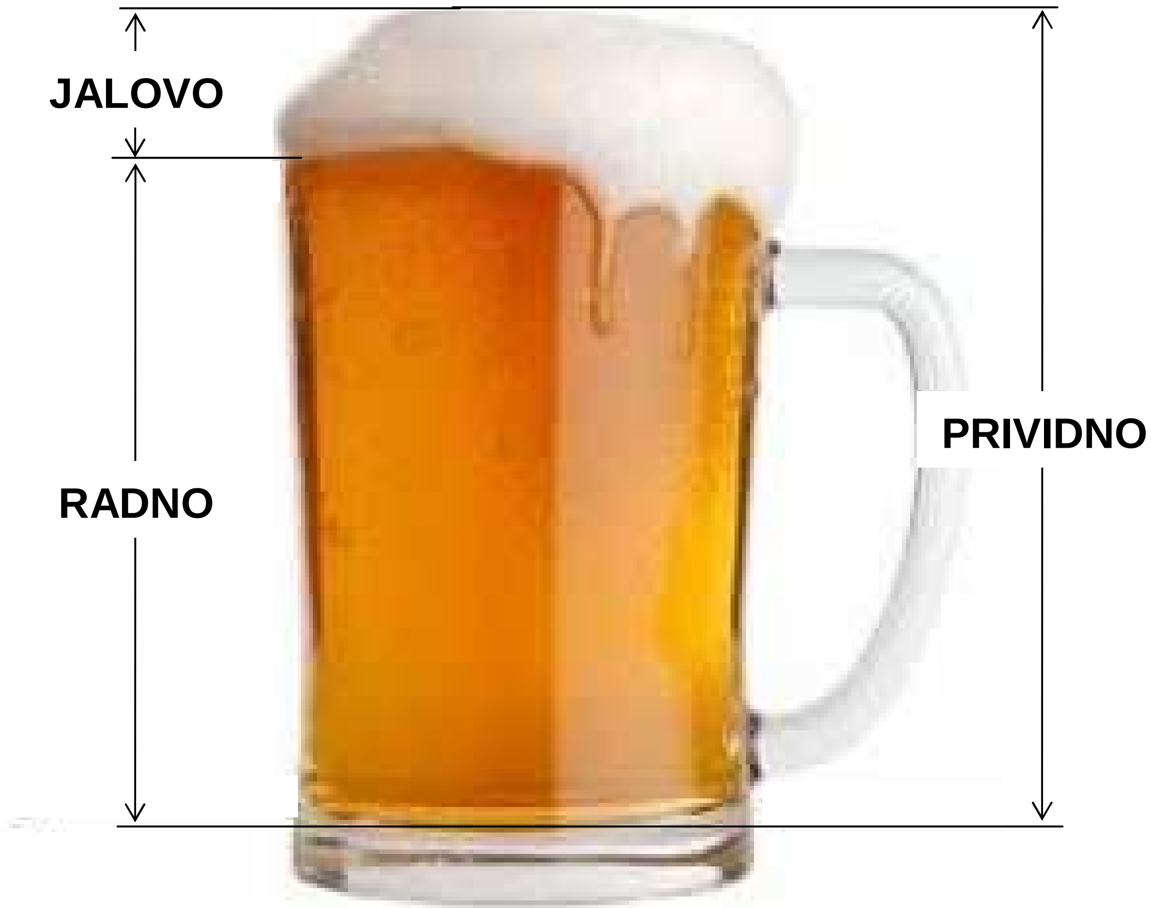


trenutne vrijednosti snage za $\varphi = 90°(\pi/2)$



Pivovara (elektrana) proizvodi pivo i naplaćuje prodavaču (elektrodistributer).

Prodavač (elektrodistributer) servira i naplaćuje žednom korisniku (potrošač).



Više pjene $\Rightarrow$ veća krigla ili više krigli za serviranje piva.

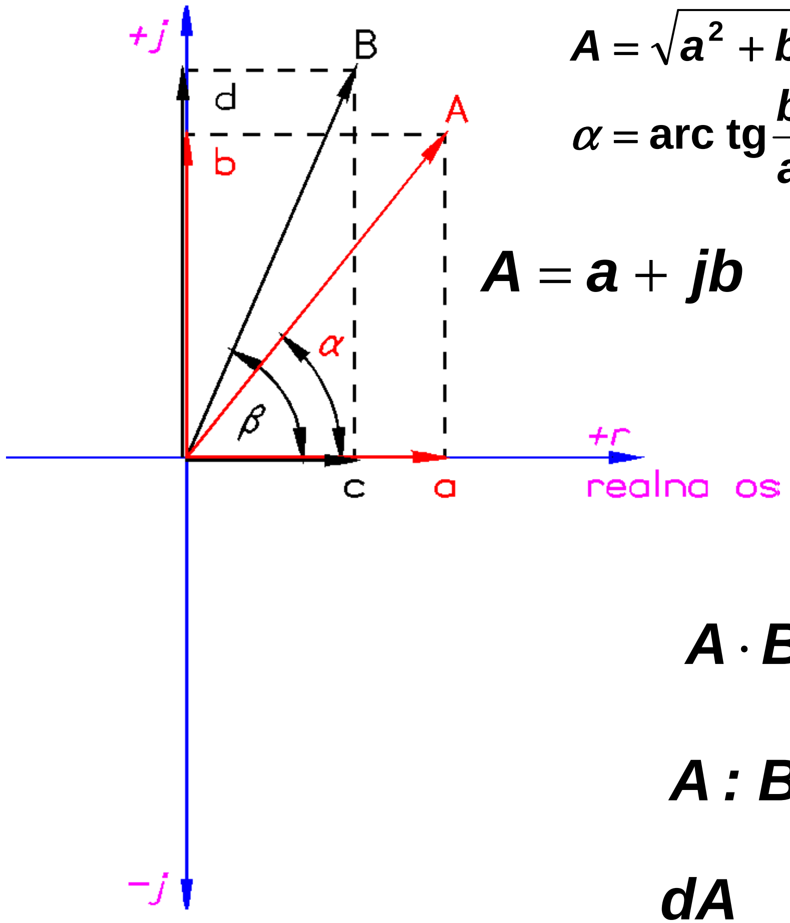
Prodavač ograničava količinu pjene ili naplaćuje serviranje pjene.

prikaz ~ struje trenutnim vrijednostima nepraktičan (vremenska domena)

prikaz u sustavu realne i imaginarne osi $\Rightarrow$ kazalo (vektor) s kutem ωt (α)

svaka periodička funkcija - vektorski zbroj realne i imaginarne komponente

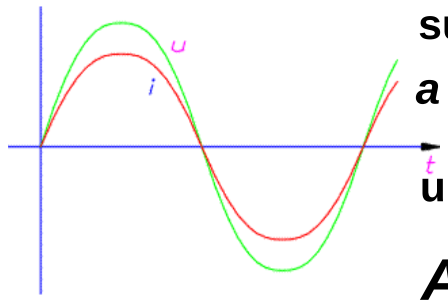
$$B = c + jd$$



$$A = \sqrt{a^2 + b^2}$$

$$\alpha = \text{arc tg}\,\frac{b}{a}$$

$$A = a + jb$$



sustav realne i imaginarne osi

$$a = A \cdot \cos\alpha \qquad b = A \cdot \sin\alpha$$

u polarnom obliku

$$\underline{A} = A \cdot e^{j\alpha} = A\angle\alpha$$

računske operacije

$$A + B = (a + c) + j(b + d)$$

$$A - B = (a - c) + j(b - d)$$

$$A \cdot B = (ac - bd) + j(ad + bc) = A \cdot B \cdot e^{j(\alpha+\beta)}$$

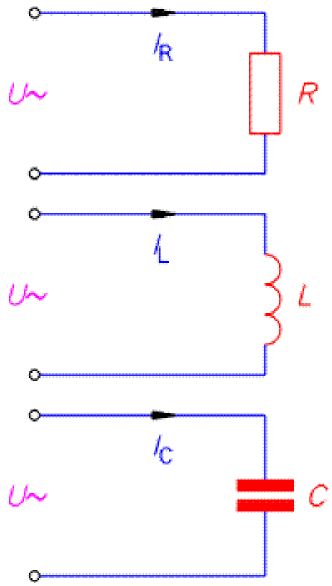
$$A : B = \frac{ac + bd}{c^2 + d^2} + j\,\frac{bc - ad}{c^2 + d^2} = \frac{A}{B} \cdot e^{j(\alpha-\beta)}$$

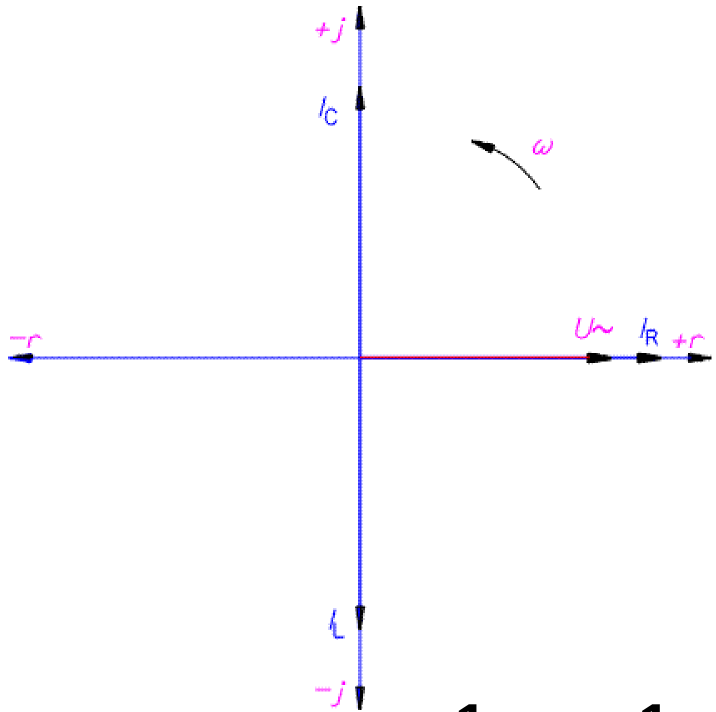
$$\frac{dA}{dt} = j\omega A \cdot e^{j(\omega t+\varphi)} \qquad \int A \cdot dt = -j\,\frac{A}{\omega} \cdot e^{j(\omega t+\varphi)}$$

omski (*R*) strujni krug $\underline{U} = U + j0 = U\angle 0 \qquad I_R = I_R + j0 = I_R\angle 0$

induktivni (*L*) strujni krug $\underline{U} = U + j0 = U\angle 0 \qquad I_L = 0 - jI_L = I_L\angle -90^o$

kapacitivni (*C*) strujni krug $\underline{U} = U + j0 = U\angle 0 \qquad I_C = 0 + jI_C = I_C\angle 90^o$





impedancija Z

$$Z = R + jX = Z\angle\varphi$$

$$Z = \sqrt{R^2 + X^2}$$

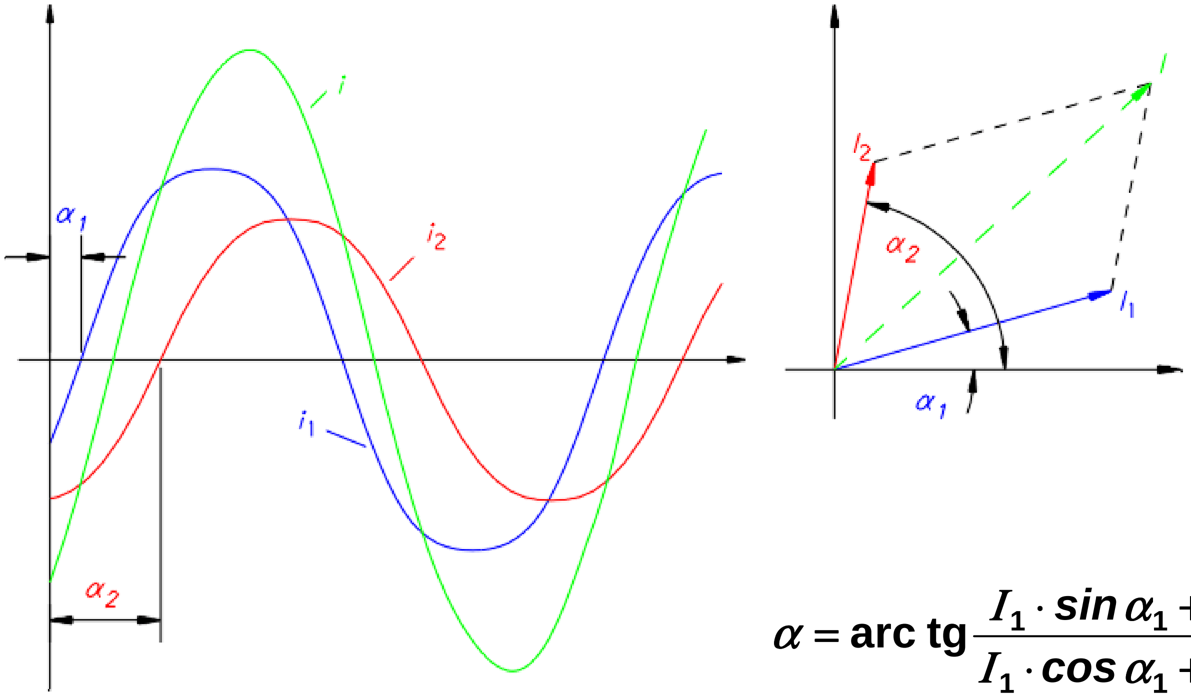
$$\varphi = \text{arc tg}\frac{X}{R}$$

admitancija Y

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$$

vrijede Ohmov $\underline{U} = \underline{I} \cdot \underline{Z}$ i Kirchhoffovi zakoni $\underline{I} = \sum_{i=1}^{n} \underline{I_i}$ $\quad 0 = \sum_{i=1}^{n} \underline{U_i}$

računske operacije sa sinusnim veličinama rezultiraju sinusnim veličinama



$$\alpha = \text{arc tg}\,\frac{I_1 \cdot \sin\alpha_1 + I_2 \cdot \sin\alpha_2}{I_1 \cdot \cos\alpha_1 + I_2 \cdot \cos\alpha_2}$$

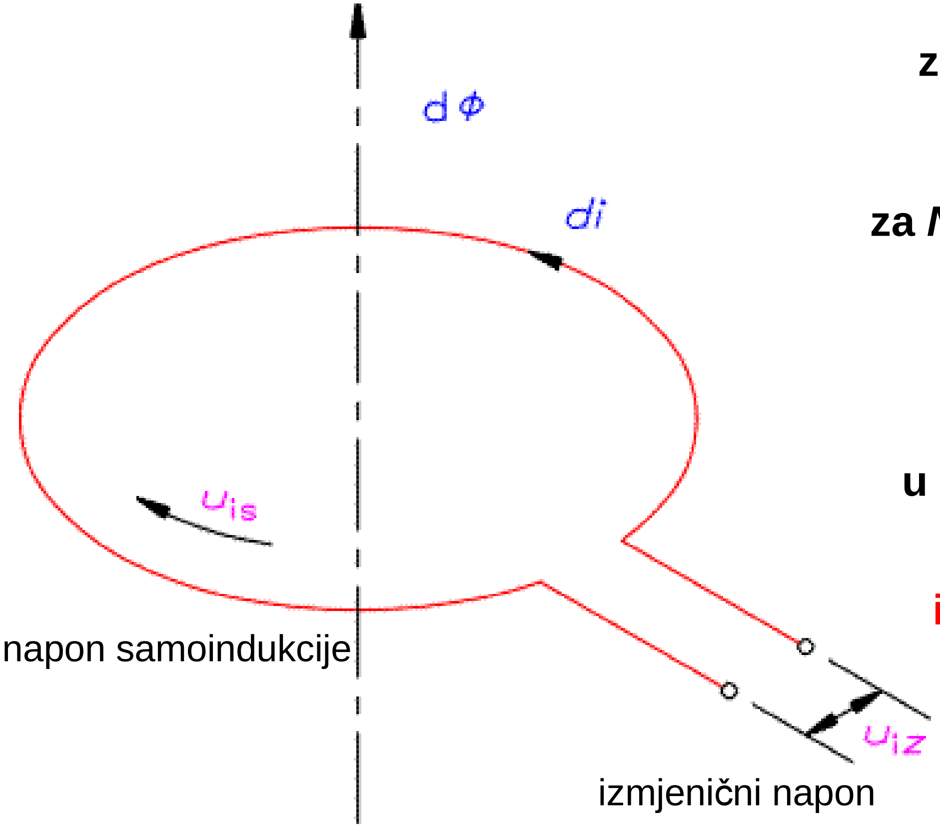
$$i_1 = \sqrt{2} \cdot I_1 \cdot \sin\left(\omega t - \alpha_1\right)$$

$$I = \sqrt{\left(I_1 \cdot \cos\alpha_1 + I_2 \cdot \cos\alpha_2\right)^2 + \left(I_1 \cdot \sin\alpha_1 + I_2 \cdot \sin\alpha_2\right)^2}$$

$$i_2 = \sqrt{2} \cdot I_2 \cdot \sin\left(\omega t - \alpha_2\right)$$

$$i = i_1 + i_2 = \sqrt{2} \cdot I_1 \cdot \sin(\omega t - \alpha_1) + \sqrt{2} \cdot I_2 \cdot \sin(\omega t - \alpha_2)$$

INDUKTIVITET U STRUJNOM KRUGU



zbog ~ $U \Rightarrow \quad u_{is} = -\frac{d\Phi}{dt}$

za N zavoja $\quad u_{is} = -N\frac{d\Phi}{dt} = -L\frac{di}{dt}$

uz $\quad u_{iz} = \sqrt{2}\cdot U \cdot \sin \omega t$

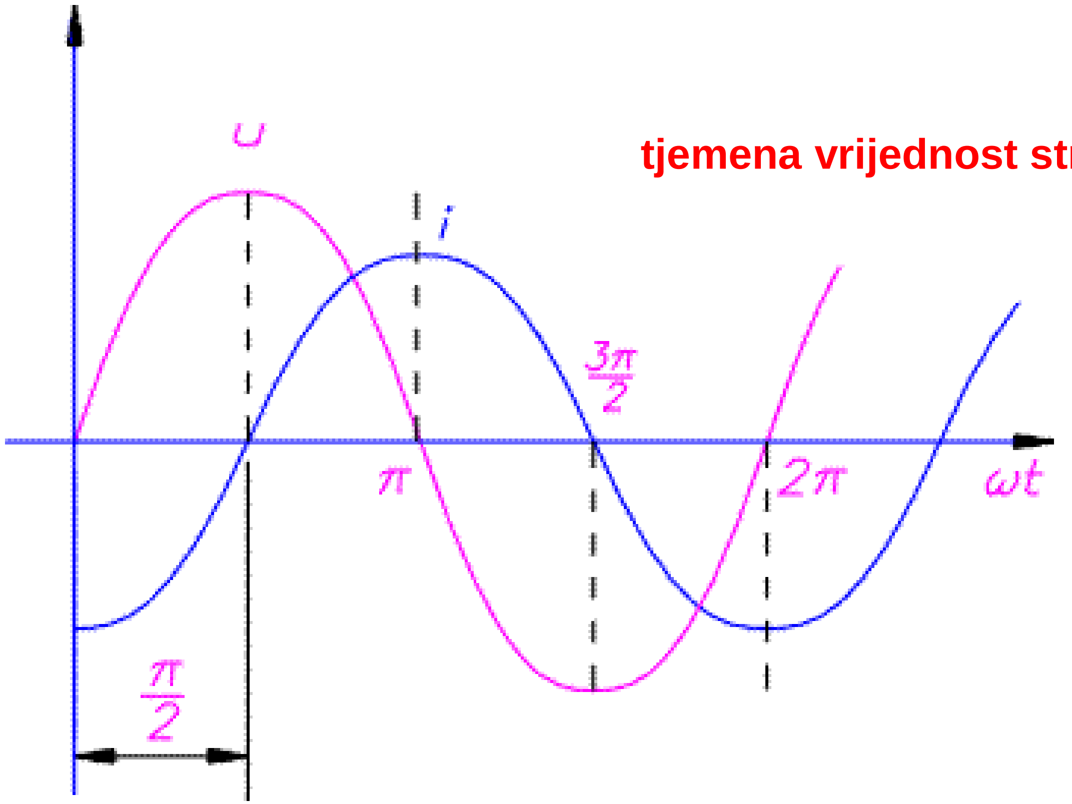
u stacionarnom stanju $\quad u_{iz} = -u_{is}$

izjednačenjem dobivamo trenutnu vrijednost *i*

$$i = \frac{\sqrt{2}\cdot U}{L}\int_0^t \sin \omega t = -\frac{\sqrt{2}\cdot U}{\omega L}\cdot \cos \omega t$$

uz $\quad -\cos \omega t = \sin\left(\omega t - \frac{\pi}{2}\right) \quad \Rightarrow \quad i = \frac{\sqrt{2}\cdot U}{\omega L}\cdot \sin\left(\omega t - \frac{\pi}{2}\right)$

struja kasni 90° za naponom



$$i = \frac{\sqrt{2} \cdot U}{\omega L} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$$

tjemena vrijednost struje

$$\sqrt{2} \cdot I = \frac{\sqrt{2} \cdot U}{\omega L}$$

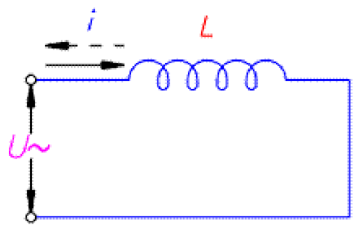
$$I = \frac{U}{\omega L}$$

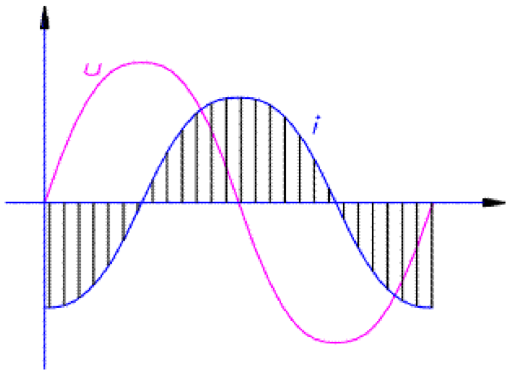
otpor induktiviteta
reaktivni otpor
INDUKTANCIJA
INDUKTIVNI OTPOR

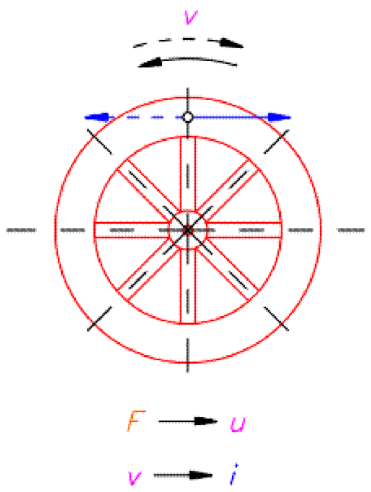
$$X_L = \omega \cdot L = 2 \cdot \pi \cdot f \cdot L$$

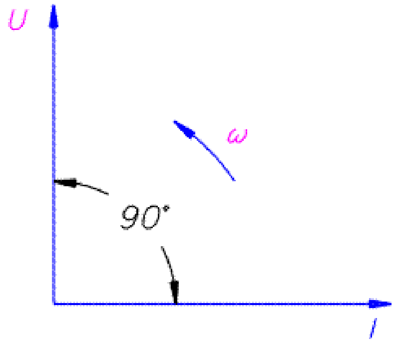
prema Ohmovom zakonu za čisti induktivitet vrijedi $\underline{U} = \underline{I} \cdot \omega L$

ponašanje induktivnog otpora u izmjeničnom strujnom krugu

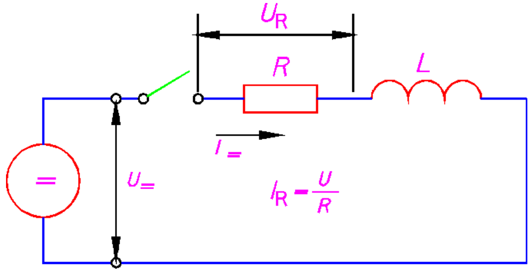


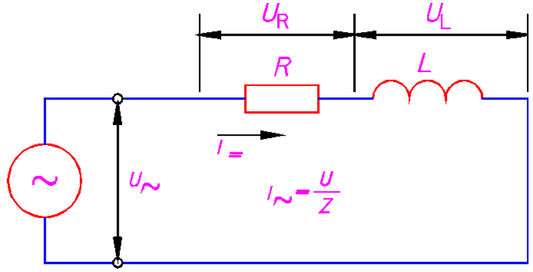


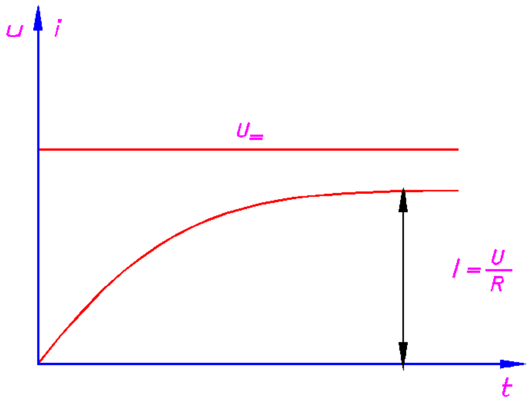


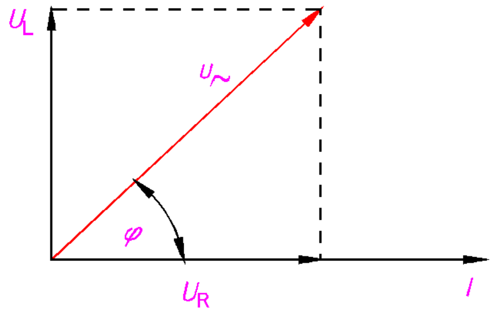


realna zavojnica (kao serijski spoj *R* i *L*) u strujnom krugu









za = strujni krug

$$I = \frac{U_=}{R}$$

za ~ strujni krug $u \Rightarrow i$ trajno uz φ

$$u_{\sim} = i \cdot R + L\frac{di}{dt}$$ ako je

$$i = \sqrt{2} \cdot I \cdot \sin \omega t$$

$$u_{\sim} = \sqrt{2}U \cdot \sin(\omega t + \varphi)$$

$$\frac{di}{dt} = \sqrt{2} \cdot I \cdot \omega \cdot \cos \omega t$$

tada vrijedi

$$U = I \cdot \sqrt{R^2 + X_L^2} = I \cdot Z$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\underline{Z} = R + jX_L = Z \cdot e^{j\varphi} = Z\angle\varphi$$

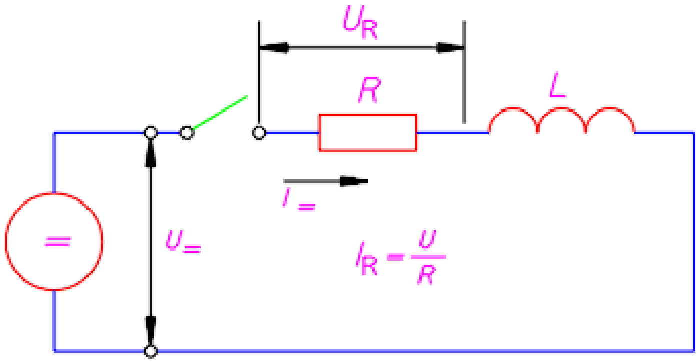
$$\varphi = \text{arc tg}\frac{X_L}{R} = \text{arc tg}\frac{\omega L}{R} \qquad 0 \le \varphi \le \frac{\pi}{2}$$

za = struju

$f=0 \Rightarrow X_L=0 \quad \varphi=0$

$\Rightarrow Z=R \Rightarrow U=IR$

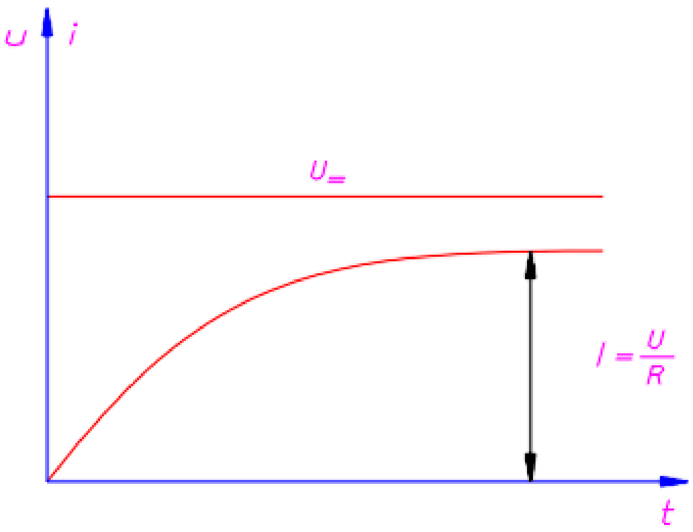
prijelazna pojava za istosmjerni napon



za $t = 0 \Rightarrow i = 0$ za $t = \infty \Rightarrow i = I = \frac{U}{R}$

uvijek vrijedi

$$U = i \cdot R + L\frac{di}{dt} \quad \text{ili} \quad \frac{dt}{L} = \frac{di}{U - i \cdot R}$$



integriranjem

$$\frac{t}{L} = -\frac{1}{R} \cdot ln(U - i \cdot R) + ln\, k$$

$$i = \frac{U}{R}\left(1 - e^{-\frac{R}{L}t}\right) = I\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\tau = \frac{L}{R}$$

$$i = I\left(1 - e^{-\frac{t}{\tau}}\right)$$

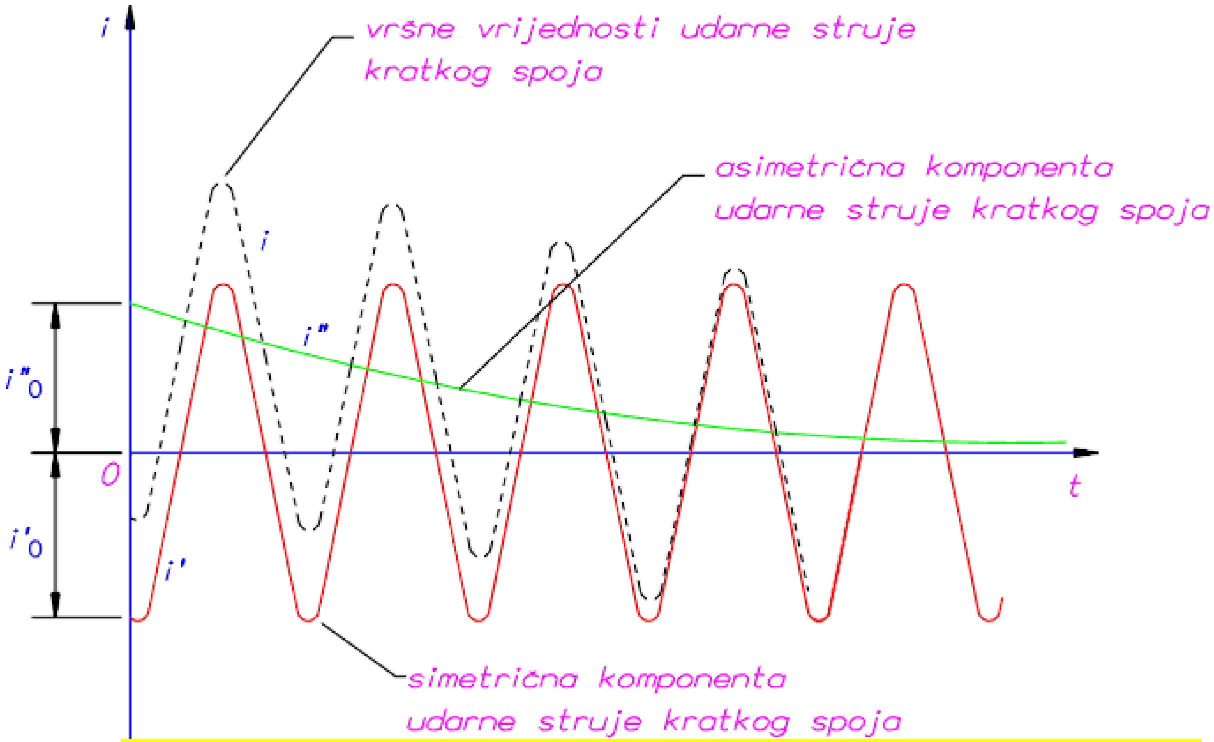
prijelazne pojave za izmjenični napon

kod priključivanja induktivnog strujnog kruga na izvor izmjeničnog napona *u* na jakost struje značajno utječe početna vrijednost izmjeničnog napona

$$u = \sqrt{2} \cdot U \cdot sin(\omega t + \alpha) = i \cdot R + L \frac{di}{dt}$$

nehomogena diferencijalna jednadžba

rješenje sadrži dvije pojave



simetričnu komponentu struje

$$i' = \frac{\sqrt{2} \cdot U}{Z} sin(\omega t + \alpha - \varphi)$$

gdje je $\varphi = \text{arc tg} \frac{X_L}{R}$

asimetričnu komponentu struje

$$i'' = \frac{\sqrt{2} \cdot U}{Z} \cdot sin(\alpha - \varphi) \cdot e^{-\frac{R}{L} t}$$

ukupna struja je

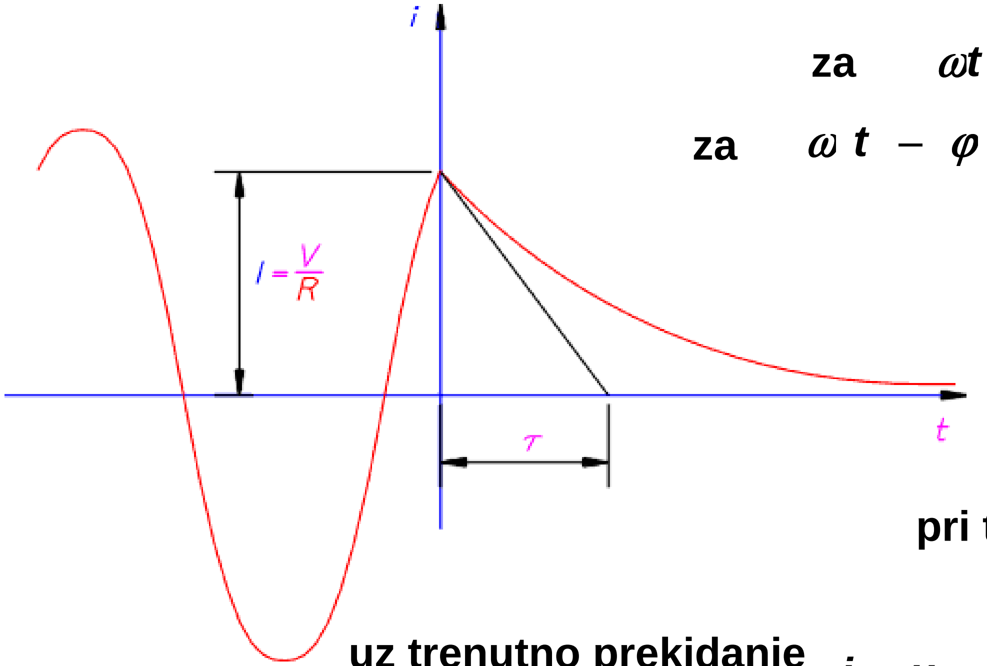
$$i = i' + i''$$

maksimalna vrijednost udarne struje kratkog spoja

$$i = 2 \cdot \frac{\sqrt{2} \cdot U}{Z}$$

pri prekidanju napajanja izmjeničnim naponom induktivnog strujnog kruga jednako ponašanje kao i pri prekidanju uz istosmjerno napajanje

mora biti zadovoljeno $i \cdot R + L\frac{di}{dt} = 0$ $\Rightarrow$ $i = \sqrt{2} \cdot I \cdot sin(\omega t - \varphi) \cdot e^{-\frac{R}{L}t}$



za $\omega t - \varphi = 0$ nema prijelazne pojave $i = 0$

za $\omega t - \varphi \neq 0$ prijelazna pojava kao za DC struju

$$i = I \cdot e^{-\frac{t}{\tau}}$$

$$i = \sqrt{2} \cdot I \cdot sin(\omega t - \varphi) \cdot e^{-\frac{t}{\tau}}$$

pri tom je $\tau = \frac{L}{R}$ vremenska konstanta

uz trenutno prekidanje toka struje ($t = 0$) $\Rightarrow$ $i_L \cdot u_L = \frac{1}{T_L} \cdot R \cdot \int i^2 \cdot dt$ $\Rightarrow$ $u_p = i_p \cdot \sqrt{\frac{L}{C}} = i_p \cdot Z_0$

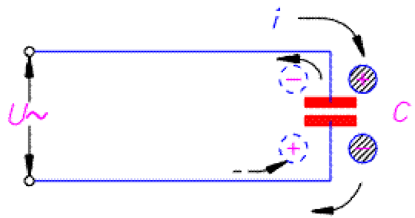
i_L - trenutna vrijednost struje
u_L - trenutna vrijednost napona
T_L - vrijeme prekidanja

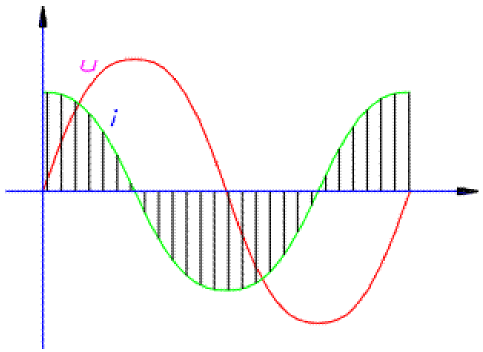
i - struja prelazne pojave
i_p - veličina prekinute struje
u_p - napon prekidanja

Z_0 - valni otpor

Z_0 =5 kΩ i_p =5 A $\Rightarrow$ u_p =25 kV

KONDENZATOR U STRUJNOM KRUGU



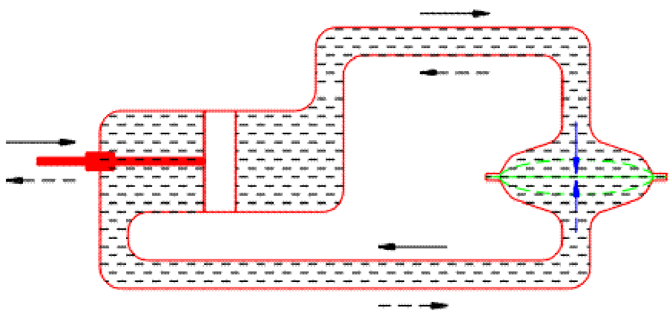


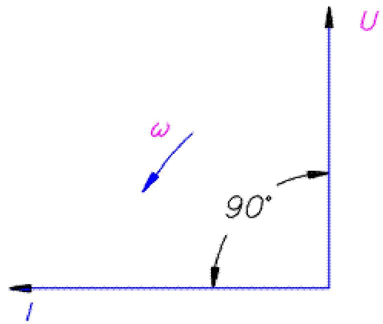
$$u = \sqrt{2} \cdot U \cdot \sin \omega t$$

$$i = \omega \cdot C \cdot \sqrt{2} \cdot U \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

tjemena vrijednost struje

struja prethodi naponu 90°





prema $I \cdot \sqrt{2} = \sqrt{2} \cdot U \cdot \omega \cdot C$

i $U = I \cdot \frac{1}{\omega C} = I \cdot X_C$

slijedi $X_C = \frac{1}{\omega C}$

kapacitivni otpor

kapacitivna reaktancija

~ napon ⇒ izmjenično nabijanje i izbijanje

⇒ promjenjivo električno polje u dielektriku

⇒ stalno prepolariziranje molekula

⇒ zagrijavanje izolatora

⇒ dielektrički gubici

gubici ⇒ ne ovise o vodljivosti izolatora (jednak karakter)

gubici ⇒ predstavljaju radnu komponentu

gubici ⇒ smanjenje faznog pomaka za kut δ

$$\varphi = \frac{\pi}{2} - \delta$$

δ **je malen te se izražava kao**

$$\mathrm{tg}\delta = \frac{I_{dg}}{I_C}$$

I_{dg} **- struja dielektričkih gubitaka**

I_C **- struja kondenzatora**

realni kondenzator (kao serijski spoj R i C) u strujnom krugu

za = strujni krug

$t = 0$

$u_C = 0 \quad i_C = \frac{U_=}{R}$

$t = \infty$

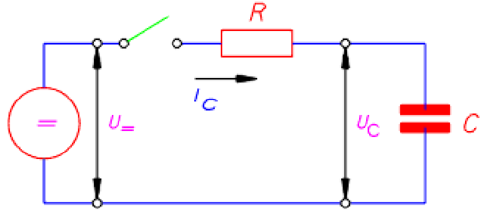
$u_C = U_= \qquad i_C = 0$

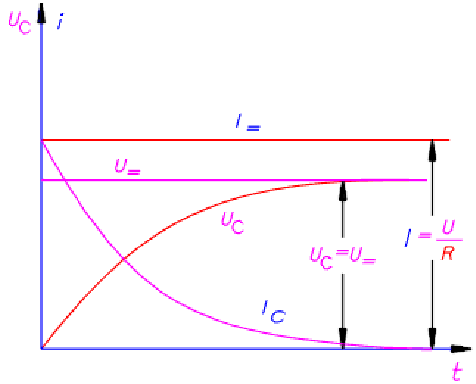
$U = i_C \cdot R + \frac{1}{C}\int i_C \cdot dt = \text{konst.}$

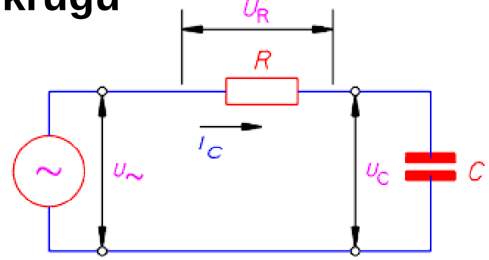
$i_C = I \cdot e^{-\frac{t}{R \cdot C}} \quad \text{uz} \quad \tau = R \cdot C$

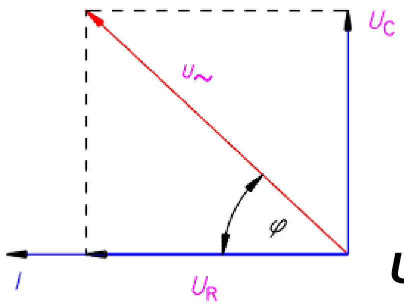
$$i_C = I \cdot e^{-\frac{t}{\tau}}$$

$$u_C = U_= \left(1 - e^{-\frac{t}{\tau}}\right)$$









za ~ strujni krug

$u \Rightarrow i$ trajno uz φ

$u = i \cdot R + \frac{1}{C}\int i \cdot dt$

ako je

$i = \sqrt{2} \cdot I \cdot \sin \omega t$

$u = \sqrt{2} \cdot U \cdot \sin(\omega t + \varphi)$

tada vrijedi

$U = I \cdot \sqrt{R^2 + X_C^2} = I \cdot Z$

$Z = \sqrt{R^2 + X_C^2}$

$\underline{Z} = R - jX_C = Z \cdot e^{-j\varphi} = Z\angle\varphi$

$\varphi = \text{arctg}\left(-\frac{X_C}{R}\right) = \text{arc tg}\left(-\frac{1}{\omega RC}\right) \quad 0 \le \varphi \le -\frac{\pi}{2}$